

where  $T$  is the kinetic energy,  $\Omega$  the gravitational energy (reckoned positive) and  $U$  the internal energy. For  $\Gamma = 4/3$ , a polytrope of index 3, the total energy is:

$$E = U - \Omega + T = -T < 0 \quad (13)$$

The system is therefore stable. With radiation pressure dominant we have:

$$\Gamma = \beta + \frac{(4 - 3\beta)(\gamma - 1)}{\beta + 12(\gamma - 1)(1 - \beta)} \simeq \frac{4}{3} + \frac{\beta}{6} \quad (14)$$

since  $\gamma = 5/3$  for stars. The total energy is then:

$$E = -\frac{\beta}{2} U - T < 0 \quad (15)$$

The reason that the total energy is now negative definite, even when  $\beta \rightarrow 0$ , is that the contribution of the rotational distortion terms to the gravitational energy, and the decrease of internal energy due to reduced pressure force, more than compensate for the added kinetic energy term.

So far as order of magnitude estimates are concerned since  $\beta$  is small, the  $U$  in equation (15) can be replaced by the unperturbed value,  $U_0$ , and the binding energy is then:

$$E = -\frac{3\beta}{4} \frac{GM^2}{R} - \frac{1}{2} I \omega^2 \quad (16)$$

where  $I$  is the moment of inertia and  $\omega$  the angular velocity. The moment of inertia can be evaluated from the theory of rotating polytropes, and for uniformly rotating polytrope of index 3, rotating with maximum possible speed (centrifugal force balancing gravity at the surface) we have:

$$E = -\frac{3\beta}{4} \frac{GM^2}{R} - 0.02 \frac{GM^2}{R} \quad (17)$$

where the number of 0.02 comes from a detailed investigation of rotating polytropes<sup>7</sup>. For massive stars  $\beta \sim 10^{-3}$  and the energy is decreased by a factor of order 30.

The stability of the objects when general relativity is included is now changed. As the rotation will only be a slight perturbation over most of the star, we can neglect rotational effects in the relativistic correction, so that the total energy is:

$$E = -\frac{3\beta}{4} \frac{GM^2}{R} - 0.02 \frac{GM^2}{R} + 5.1 \frac{GM^2}{R} \left( \frac{GM}{Rc^2} \right) \quad (18)$$

The massive star will become unstable when  $\partial E / \partial R = 0$  which gives:

$$R_I \simeq 250 R_g \quad (19)$$

independent of the mass of the object provided  $\beta$  is small.

Now the radius of main sequence massive objects is given from the structure of the star as<sup>8</sup>:

$$R_{MS} = 1.5 \times 10^{11} \left( \frac{M}{M_\odot} \right)^{1/2} \text{ cm} \quad (20)$$

This is slightly larger than the non-rotating value due to the expansion of the equatorial regions by the centrifugal force<sup>9</sup>. In terms of the Schwarzschild radius, this is:

$$R_{MS} = 7.5 \times 10^5 R_g \left( \frac{M}{M_\odot} \right)^{-1/2} \quad (21)$$

Hence we have:

$$\frac{R_{MS}}{R_I} = 3 \times 10^3 \left( \frac{M}{M_\odot} \right)^{-1/2} \quad (22)$$

This brings the instability right into the interesting region where  $M \simeq 10^7 M_\odot$ . Stars smaller than this can reach the main sequence configuration, burn hydrogen and then collapse.

The rotation will also affect the time-scale of the Hayashi type evolution prior to hydrogen burning or gravitational collapse. With a luminosity:

$$L \sim 5 \times 10^4 \left( \frac{M}{M_\odot} \right) L_\odot \quad (23)$$

which draws on the internal energy, the time required to contract to the radius of instability,  $\tau$ , satisfies:

$$L\tau = -E = \frac{1}{100} \frac{GM^2}{R_I} \quad (24)$$

which gives:

$$\tau \simeq 10^4 \text{ years} \quad (25)$$

for any mass. This estimate is not accurate, as we have neglected the energy carried away by mass loss during contraction. This will reduce the life-time by a factor of two<sup>9</sup>.

If the star is rotating non-uniformly, with the central regions rotating faster than the surface layers, this will aid stability. An inward increase in  $\omega$  by a factor of 6 would permit a star of  $10^{10} M_\odot$  to reach the hydrogen-burning phase. Such an inward increase may be possible in a convective star with very large turbulent velocities where each turbulent eddy is conserving its angular momentum. This possibility will be considered elsewhere<sup>9</sup>, as it requires an examination of the convective energy transport in massive objects and is outside the scope of the present article.

It should be noted that the stabilizing effect of rotation is due to the presence of  $2T$  in the virial theorem. Consequently any other form of kinetic energy, such as that in the turbulent convection, could also exert a stabilizing influence.

This investigation arose out of discussions with Prof. W. Fowler at the Second Texas Symposium on Relativistic Astrophysics held at the University of Texas, Austin.

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## COOLING OF NEUTRON STARS

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THE discovery of discrete sources of X-rays in the sky has led to much speculation as to the mechanisms responsible for the X-ray emission<sup>1-6</sup>. Among these suggestions has been the thermal emission of X-rays from the hot surfaces of neutron star remnants of supernova explosions<sup>1-3</sup>. Calculations of the cooling of such a neutron star following its formation have seemed to indicate that

the temperature would still be several million degrees at an age of several thousand years. Such cooling calculations have been based on estimates of photon emission from the surface and of neutrino-antineutrino pair emission by the plasma process from the interior. There have recently been new suggestions regarding other possible mechanisms of neutrino emission from the interior<sup>7</sup>, and this has led

to the impression that neutron stars might cool too rapidly to be observable as celestial X-ray objects<sup>5</sup>. We believe this view to be too pessimistic, and we give here a brief summary of some of our cooling calculations for neutron stars.

The neutron star models we have used in our calculations have been based on composite equations of state with nuclear forces included. The details of the construction of the equations of state have been described in a thesis<sup>3</sup>, and will be published in due course. At the lowest densities, but still at substantially high temperatures, the composition was assumed to be iron. As the density increases, the main contribution to the pressure comes from the degenerate electrons, and the mean molecular weight per electron gradually increases as the high Fermi level of the electrons forces the nuclear composition of the material to change toward higher mass number. At densities between  $10^{11}$  and  $10^{14}$  g/cm<sup>3</sup> the heavy nuclei gradually dissolve into neutrons. The system then consists of degenerate neutrons, protons and electrons. Near and above  $10^{15}$  g/cm<sup>3</sup>, many other particles appear in the mixture, commencing with the  $\mu$  meson and the  $\Sigma^-$  hyperon. In constructing our equations of state, the degeneracy pressures of all the individual fermions were added, and nuclear force terms were also included.

Nuclear forces are generally attractive at large internucleon distances and repulsive at small internucleon distances. In this work we chose forms of the potential interactions between neutrons suggested by Levinger and Simmons<sup>8</sup>. Their potentials  $V_\beta$  and  $V_\gamma$  were utilized. Both these potentials are attractive at low densities, although  $V_\gamma$  is somewhat more attractive than  $V_\beta$ . At high densities  $V_\gamma$  rapidly turns repulsive, while  $V_\beta$  only slowly turns repulsive. For our neutron star equations of state, we have applied these potentials between the baryons without distinction as to the type of baryon. At densities less than, or equal to, nuclear densities, the character of nuclear forces is reasonably well known, and is given to a rough approximation by either of these potentials; and the composition of the matter is mostly neutrons, for which the potentials were originally constructed. At much greater than ordinary nuclear densities many different types of baryon are present, and the

rapidity with which nuclear forces turn repulsive is very speculative. Hence the two potentials  $V_\beta$  and  $V_\gamma$  tend to span a range of possible behaviour of the nuclear forces at high densities, and the differences in the neutron star models which result from the adoption of one or the other potential will give an indication of the uncertainty due to lack of knowledge in this area of physics.

In the case of a perfect fluid there is a general relativistic limitation on the pressure such that it cannot exceed one-third of the proper energy density<sup>9</sup>. In a fluid with suitable anisotropic properties, this condition may be violated; but the ultimate relativistic condition still remains that the pressure cannot exceed the proper energy density<sup>10</sup>. If this were to be violated the speed of sound would exceed the speed of light in the medium. Accordingly, the composite equation of state constructed as already described was cut off with one of these two pressure saturation conditions.

The general relativistic equations of hydrostatic equilibrium were derived by Oppenheimer and Volkoff<sup>11</sup>. For a given assumption about central density, the numerical integration of the differential equations of hydrostatic equilibrium gives models with uniquely determined masses. The gravitational and proper masses determined in this way as a function of central density for the two composite equations of state are shown in Fig. 1. Several interesting comments follow from this figure. It is evident that the details of the structure of the neutron star models are very sensitive to the uncertainties in the rates at which nuclear repulsive forces enter in the baryon mixture at high densities. For each equation of state the mass rises with increasing central density toward a principal peak, beyond which it falls and then oscillates. It is interesting to note that the gravitational mass is less than the proper mass over the entire range of neutron star models, thus showing that such models are gravitationally bound. This is contrary to the behaviour of models constructed with non-interacting neutron gases. A more detailed discussion of the stability of the models beyond the main peak will be given separately<sup>12,13</sup>. It may also be noted that pressure saturation does not set in until the vicinity of the principal peak has been reached. Hence uncertainties in the pressure saturation effects play no significant part in the discussion which follows.

In order to calculate representative cooling curves for neutron star models, three models were chosen so that one was of low mass at the low-density base of the principal peak, one was half-way up the peak, and the other was near the top of the peak. To these models were fitted hot atmospheres corresponding to a series of surface temperatures. These atmospheres were composed of both iron and magnesium; the results were nearly the same, and only the neutron stars with iron envelopes are discussed here. It was typically found that the temperature rose by about a factor of 50 between the photosphere of the neutron star and the interior where the high thermal conductivity of the electrons assured a flat temperature distribution. The atmospheres were constructed with the help of opacities calculated from the Los Alamos opacity code of A. N. Cox *et al.* The atmospheric structure was determined by requiring that the luminosity should not change from one layer to the next.

The heat capacity of the neutron star models is a function of their temperature<sup>14</sup>. The presence of nuclear forces in the equation of state will modify the heat capacity by an amount which typically can be of the order of a factor 2, according to rough estimates which we have made. We did not take such modifications into account in making the actual cooling calculations.

The cooling of the neutron stars is due to the combination of neutrino emission from the interior and photon emission from the surface. The construction of the envelope automatically provided us with the photon cooling rates. Three neutrino cooling rates were taken into account, as follows:

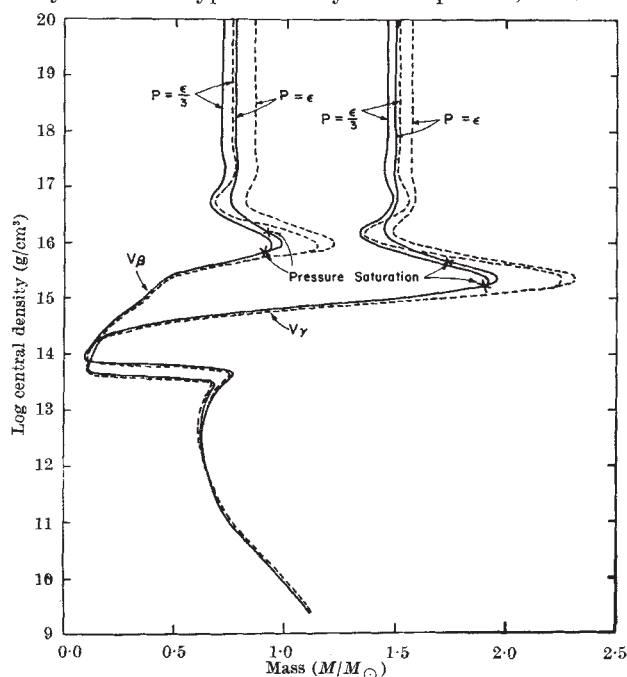


Fig. 1. Gravitational (—) and proper (---) masses of the neutron star models constructed with both composite equations of state and plotted as a function of central density. The different models resulting from the two pressure saturation conditions at the highest densities are separately indicated.

(1) *Neutrino pair emission from the plasma process.* These neutrinos arise from the decay of plasmons in the degenerate electron gas in the interior of the neutron star. The rates have been given by Adams, Ruderman and Woo<sup>15</sup> and by Inman and Ruderman<sup>16</sup>.

(2) *The URCA process.* Bahcall and Wolf have recently given an estimate of neutrino emission from this process<sup>7</sup> in which neutrons decay into protons and protons capture electrons. The rate is somewhat greater than that of the plasma process.

(3) *The neutrino bremsstrahlung process.* The neutrino pairs are emitted when electrons scatter from positive or negative baryons in the interior of the neutron star. M. A. Ruderman and G. Festa (private communication) have kindly provided us with the following approximate preliminary expression for this process:

$$q \left( \frac{\text{ergs}}{\text{g-sec}} \right) = 10^6 Z^2 \frac{n_z}{n} (T_9)^6 \quad \text{for } E_F \gg mc^2$$

where  $Z$  is the effective charge of the electron scattering centres,  $n_z$  is the number density of such centres, and  $n$  we take here to be the baryon number density. Ruderman and Festa have suggested that there may be proton clustering in neutron star interiors in the presence of very large numbers of neutrons, so that the effective charge of scattering centre might be 2. However, it may well be that under conditions of interest in the interiors of neutron stars there will also be a large number of  $\Sigma^-$  hyperons, which may hinder the clustering process. Consequently, we have chosen to take the effective charge of a scattering centre equal to unity and to count as the scattering centres both the protons and the  $\Sigma^-$  hyperons. This process is less important than the URCA process at high temperatures, but it is more important at low temperatures.

The cooling curves for the six chosen neutron star models as a function of age are shown in Fig. 2. It may be seen that the rate of cooling has a significant dependence on the mass of the star. The low-mass stars cool quite rapidly, but the medium and heavy mass stars still have temperatures exceeding  $2 \times 10^6$  °K for times of the order of  $10^4$  or  $10^5$  years. Hence it is evident that thermal emission of X-rays from neutron star surfaces should continue to be regarded as candidates for identification with some of the X-ray sources which are being found in the sky.

Bahcall and Wolf<sup>7</sup> have raised the question of neutrino cooling from pion decays in neutron star interiors. Such pion decays can occur only if pions should have a small effective mass in the presence of a largely neutron gas. Both Bahcall and Ruderman have indicated to us (private communications) their expectations that, under the conditions in which pions may be present in a neutron star, there will be a predominantly repulsive interaction between the pions and the neutrons. This would raise, rather than lower, the effective mass of the pions, and make it very unlikely that pions will be present in the interiors of neutron stars on the low-density side of the principal peak.

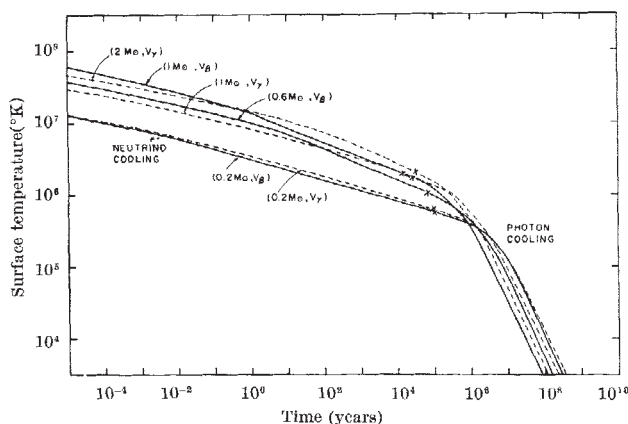


Fig. 2. The cooling curves for selected neutron star models fitted with atmospheres of iron, with three models for each composite equation of state. In the portions of the curves to the left of the crosses cooling occurs predominantly by neutrino emission from the interior; to the right of the crosses photon cooling from the surface predominates. —,  $V_F$ ; ---,  $V_F$ ; ×, cooling rates equal.

The remarkable NRL rocket experiment carried out during a lunar occultation of the Crab Nebula has shown that the X-ray source associated with the Crab Nebula has dimensions very much larger than those of a neutron star. One of us has suggested that these X-rays may be due to the synchrotron process from high energy electrons accelerated in the magnetosphere of a vibrating neutron star<sup>8</sup>. Similar non-thermal processes may well be associated with other X-ray sources, if they are neutron stars, and hence non-thermal components to X-ray spectra may be common. Hence we believe that many more highly refined experiments will be necessary before the true nature of the celestial X-ray sources will be determined.

We thank Dr. M. A. Ruderman and Mr. G. Festa for communicating to us in advance of publication their preliminary results on neutrino emission by the bremsstrahlung process.

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## INTERSTELLAR EXTINCTION BY GRAPHITE GRAINS

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MANY observational data have recently accumulated on the law of interstellar extinction in the galaxy<sup>1-3</sup>. The general trend of the new results appears to be a considerable variability in the extinction law. The 'unique extinction law' hitherto believed to be generally valid now appears to hold only in limited regions of the sky. The results of a recent investigation by Johnson<sup>3</sup> indicate different extinction curves for Cygnus, Orion,

Perseus, Cepheus and NGC 2244 (Fig. 1). Dr. K. Nandy<sup>12</sup> also obtains different extinction curves for Cygnus and Perseus. The observations of Johnson<sup>3</sup> are normalized to give an extinction of 1 mag. at  $\lambda = 5470$  Å.

From a theoretical point of view the present situation is in fact quite satisfactory. To understand a unique extinction law one has to impose very stringent conditions on the sizes of interstellar particles. For a physically